SS_Fitting

Theory and User Manual

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CONTENTS

INTRODUCTION

The hydrodynamic module of FAST, called HydroDyn, includes the contribution of wave radiation forces. The free surface memory effects that are part of the wave radiation forces are modeled using a time convolution of the retardation function. Instead of using this method, one could fit a state-space model to the retardation function in order to compute the radiation forces. This new approach will enable both a loose and tight coupling of the hydrodynamic forces within the new FAST modularization framework. In particular, the tight coupling scheme has the capability to provide better numerical accuracy and stability of the model than the loose coupling scheme. In addition, it can enable the linearization of the complete aero-hydro-servo-elastic solution, including the wave-radiation forces, which is useful for eigenanalysis and the development of new control algorithms for floating wind systems. Linearization of the convolution method is also possible if the convolution is implemented numerically in discrete time, but then the resulting linearized system must include a combination of continuous and discrete time states. See Jonkman (2013) for more information.

SS_fitting is designed to provide a state-space model based on the WAMIT output files. These matrices will be required to use the new State-Space realization module of HydroDyn, in order to compute the radiation forces of a floating wind turbine.
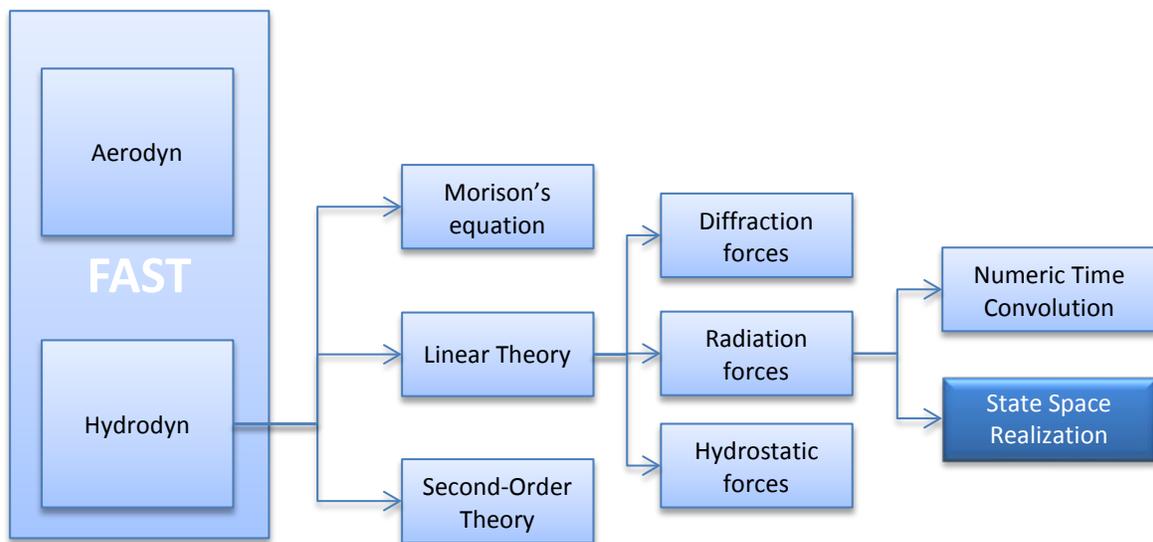
FIGURE 1: HYDRODYN MODULES INCLUDING THE NEW STATE-SPACE REALIZATION MODULE.

LINEAR HYDRODYNAMIC THEORY

The hydrodynamic forces applied to a free floating body can be described by the application of the second *Newton's Law*, for each degree of freedom i,

$$M\ddot{q} = F^{hydrostatic} + F^{radiation} + F^{Waves}$$

1

Where the M is the body mass matrix of the floating body, q the displacement vector and F represent the different hydrodynamic forces acting on the body. These include the hydrostatic restitution forces, $F^{hydrostatic}$; added mass and damping from the radiation problem including free-surface memory effects, $F^{radiation}$ and excitation forces from incoming waves, F^{waves}.

These forces described are due to the external pressure acting on the body. The hydrostatic term represents the restoring force due to gravity and buoyance. In the linear theory, under the assumption of small motion of the device and waves, this term is linear and proportional to the displacement of the body, simply given by:

$$F^{hydrostatic} = -C^{hydrostatic}q + \delta_{i3}\rho V_0 g \tag{2}$$

Where $-C^{hydrostatic}$ is the hydrostatic restitution matrix and q is the body displacement. The second term represents the impulse force in the body undisturbed position, where δ_{i3} is the Dirac delta, ρ is the water density, V_0 is the submerged volume in the undisturbed position and g is the acceleration of gravity. The radiation forces arise from the change in momentum of the fluid due to the motion of the structure. Under the linear wave approximation the radiation force in an ideal fluid can be represented by:

$$F^{radiation} = -A_\infty \ddot{q} - \int_0^t K\ (t-\tau)\dot{q}(\tau)\delta\tau \tag{3}$$

The first term represents the added mass forces associated with the fluid displaced due to the acceleration of the device, in which A_∞ is the constant positive infinite-frequency added mass matrix. The second term represents the fluid memory effects that incorporate the energy dissipation due to the radiated waves generated by the motion of the body. This term is represented by the time convolution of the body velocities and the radiation impulse-response function, $K(t)$, also called the *retardation* or *memory* matrix. This is not a very efficient term to compute numerically, as it requires information from previous time steps, in theory from the start of the body motion. Most of the codes using this formulation truncate the integral in equation 4:

$$F^{radiation} = -A_\infty \ddot{q} - \int_{t-t_{memory}}^t K(t-\tau)\dot{q}\delta\tau \tag{4}$$

Storing only a few seconds of 'memory' (t_{memory}), usually 60 s. The accuracy of this method depends on the amount of time stored (which increases the computational time), and the quality of the impulse-response function of the platform modeled.

Substituting equations 2 and 3 in equation 1, we obtained the so called *Cummins Equation* (Cummins, 1962):

$$(M + A_\infty)\ddot{q}\ + \int_0^t K(t-\tau)\dot{q}\delta\tau + C^{hydrostatic}q = F^{waves} \tag{5}$$

HydroDyn actually uses a slightly different version of 4, including the effects of drag forces to viscous effects, $F_i^{viscous}$ and the mooring lines restitution, $F_i^{moorings}$:

$$(M + A_\infty)\ddot{q}\ + \int_0^t K(t-\tau)\dot{q}\ \delta\tau + C^{hydrostatic}q\ - \rho g V_0 \delta_{33} = F^{waves} + F^{viscous} + F^{moorings} \tag{6}$$

The term $-\rho g V_0 \delta_{33}$ represents the buoyancy force at the platform undisturbed position, which is canceled by the gravity force acting on the platform and turbine. This equation is implemented in HydroDyn according to Figure 2.

As seen in the figure, the coefficients A_∞, K and $C^{hydrostatic}$ must be computed by a hydrodynamic 3D panel code, like WAMIT, which provide the frequency-dependent added mass and damping matrices, as well as the hydrostatic matrix and wave-excitation forces.

As seen all the terms of equation 5 are easily numerically implemented, except the time convolution term. This is not a very efficient term to compute from the numeric point of view, and it also constrains the linearization of the wind turbine model. However, it has been proposed in the literature several approaches to fit a parametric model to the behavior of the memory function. This is highlighted in the next chapter.
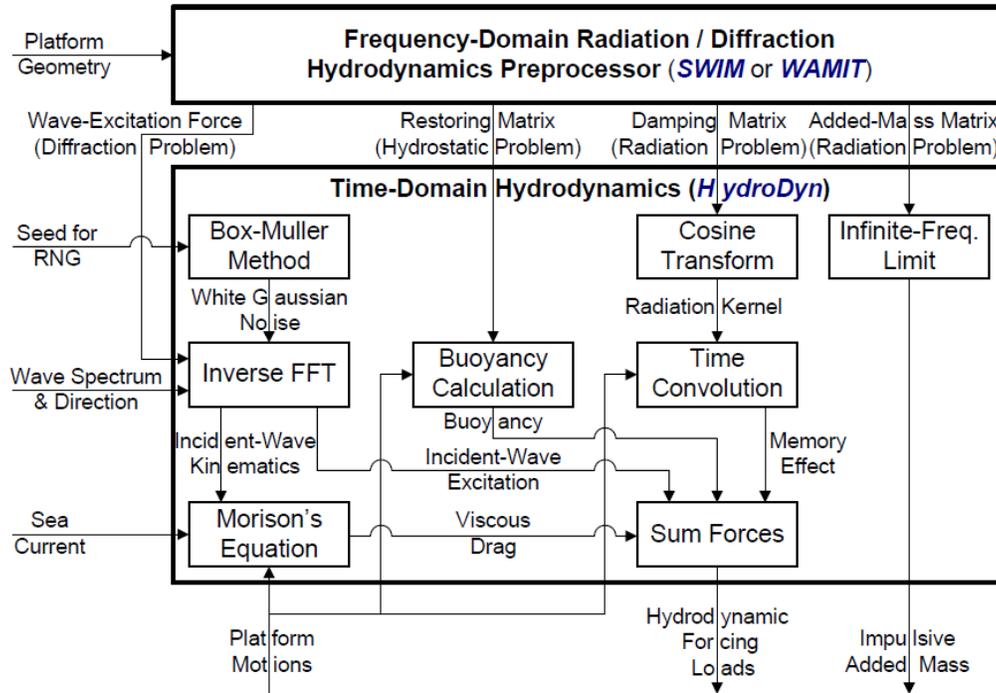
FIGURE 2: SUMMARY OF THE CALCULATIONS FOR THE INTERFACE OF SUPPORT PLATFORM LOADS TO FAST AND ADAMS. (JONKMAN & BUHL, 2007)

RADIATION FORCE AND FREQUENCY DEPENDENT PARAMETERS

The convolution integral of the radiation force becomes in the frequency domain just the multiplication of the *Fourier* transform of the impulse response function $K(t)$ by the body velocity $\dot{q}$. The retardation function $K(\omega)$ may be decompose into real and imaginary parts:

$$K(\omega) = B(\omega) + j\omega[A(\omega) - A_\infty]$$ **7**

The coefficients $A(\omega)$ and A_∞ represent the frequency-dependent added mass and the infinite-frequency added mass respectively. The term $B(\omega)$ represents the frequency-dependent damping matrix and j the imaginary unit. As seen before, all these coefficients may be obtained from a hydrodynamic panel code.

The relation between these parameters and the impulse-response function was derived by Ogilvie (Ogilvie, 1964) via a direction application of the *Fourier* transform under a sinusoidal regime:

$$A(\omega) = A_\infty - \frac{1}{\omega} \int_0^\infty K(t) \sin(\omega t)\, dt$$ **8**

$$B(\omega) = \int_0^\infty K(t) \cos(\omega t)\, dt$$ **9**

Where it follows that $A(\infty) = \lim_{\omega \to \infty} A(\omega)$. Given the frequency-dependent damping matrix, it is possible to compute the impulse-response function using:

$$K(t) = \frac{2}{\pi} \int_0^\infty B(\omega) \cos(\omega t)\, d\omega$$ **10**

The impulse-response function can be obtained either using 10 or the inverse Fourier transform of equation 7.

PARAMETRIC MODEL IDENTIFICATION OF THE CONVOLUTION INTEGRAL

In order to compute more efficiently the free-surface memory effects, it is possible to fit a parametric model to approximate the convolution term in the Cummins equation. This can be done, assuming that the system is causal and time invariant, using a state-space model described by:

$$\mu = \int_0^t K(t - \tau)\dot{q}\delta\tau \cong \left| \begin{matrix} \dot{x}_r = A_r x_r + B_r \dot{q} \\ \mu = C_r x_r \end{matrix} \right. \tag{11}$$

This process involves the identification of the state-space system with matrices A_r, B_r and C_r. One of the advantages is the Markovian propriety of the state-space models, which guarantees that any future state of the system depends only on the present value of the system states. In other words, no past information needs to be stored as in the case of the convolution method, as all the memory effect is contained in the state vector x_r.

Several methods have been proposed in the literature to perform this system identification, see for example (Jefferys, et al., 1984), (Jefferys & Goheen, 1992), (Yu & Falnes, 1995), (Yu & Falnes, 1998), (Holappa & Falzarano, 1999), (Hjulstad, et al., 2004), (Kristansen & Egeland, 2003), (Kristiansen, et al., 2005), (Jordan & Beltran-Aguedo, 2004), (McCabe, et al., 2005) and (Perez & Fossen, 2009).

In order to obtain the state-space system represented by equation 11 it is necessary to compute the frequency-dependent added mass and damping matrices using a numerical code, as seen before. The retardation function in the frequency domain is straightly forward compute using equation 7, and with the frequency-response it is possible to find the equivalent linear state-space model. This method is named *Frequency-Domain Identification*. However, using the inverse *Fourier Transform* of the retardation function or equation 10 directly, it is possible to find the impulse-response function of retardation function. Using this *Time-Domain Identification*, it is possible to find the state-space model with the equivalent impulse-response. These methods are summarized in Figure 3.
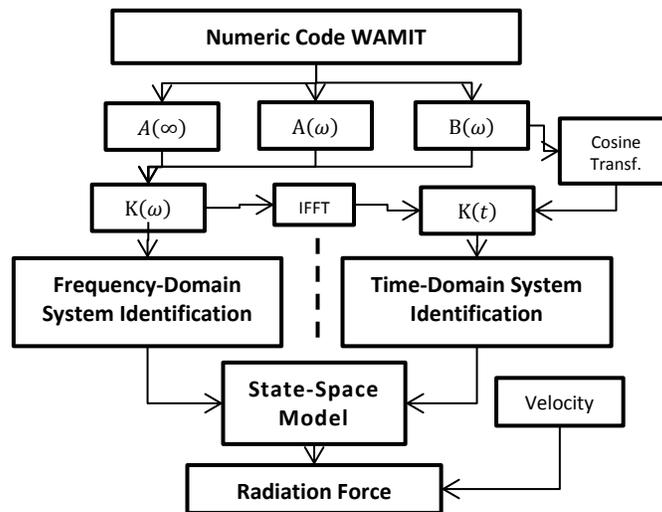

FIGURE 3: SCHEME OF THE RADIATION FORCE CALCULATION USING FREQUENCY- OR TIME-DOMAIN IDENTIFICATION METHODS.

Based on the literature, four different methods were implemented within the toolbox. The user can choose the one to used, defining it in the input file (see section User Manual). The different methods are described in the following sections.

PROPRIETIES OF THE PARAMETRIC MODELS

One can fit a parametric model (transfer function) with the appropriate order for each entry of the retardation matrix:

$$\widetilde{K}_{ij}(s,\theta) = \frac{P(s,\theta)}{Q(s,\theta)} = \frac{p_m s^m + p_{m-1}s^{m-1} + \cdots + p_0}{s^n + q_{n-1}s^{n-1} + \cdots + q_0} \tag{12}$$

Where $\theta = [p_m, \ldots, p_0, q_{n-1}, \ldots, q_0]$ is the vector with the different parameters of the numerator $P(s,\theta)$ and denominator $Q(s,\theta)$, $\widetilde{K}_{ij}$ is an entry of the retardation matrix and $s = j\omega$. The parametric models fitted to the retardation function should fulfill certain proprieties known apriori. These were derived using the hydrodynamic proprieties of the radiation potential described in Perez & Fossen, 2008 and are summarized in Table 1 at the end of the section.

LOW-FREQUENCY ASYMPTOTIC VALUE

The low-frequency asymptotic value is given by:

$$\lim_{\omega \to 0} K(\omega) = 0 \qquad \textbf{13}$$

This statement is based on the principle that any structure cannot radiate waves at zero-frequency. If the retardation function is approximated by equation 10, then the function has to have a zero at $\omega = 0$. This means that the parameter p_0 has to be zero.

HIGH-FREQUENCY ASYMPTOTIC VALUE

The high-frequency limit of the retardation function has to be 0:

$$\lim_{\omega \to \infty} K(\omega) = 0 \qquad \textbf{14}$$

This is can be proved based in equation 7. The damping limit has to be zero as the structure cannot radiate waves with infinite frequency, and so the difference $A_{ij}(\omega) - A_\infty$ will tend to zero when $\omega \to \infty$ (see Falnes, 2002 for more details).

To guarantee this propriety, the transfer function $K(j\omega)$ has to be strictly proper, that is $\deg\{Q(s,\theta)\} > \deg\{P(s,\theta)\}$. This will guarantee that the denominator grows faster with ω than the numerator, and therefore the function will tend to 0 when the frequency tends to infinite.

INITIAL TIME VALUE

The impulse-response function of the retardation function must have initial value different from zero. This can be proven by equation 10:

$$\lim_{t \to 0} K_{ij}(t) = \lim_{t \to 0} \frac{2}{\pi} \int_0^\infty B(\omega) \cos(\omega t) \, d\omega = \frac{2}{\pi} \int_0^\infty B(\omega) d\omega \neq 0 \qquad \textbf{15}$$

Applying the Laplace transformation to equation 15:

$$\lim_{t \to 0} K_{ij}(t) = \lim_{s \to \infty} s K_{ij}(s) = \lim_{s \to \infty} s \frac{P(s)}{Q(s)} = \frac{p_m s^{m+1}}{s^n} \qquad \textbf{16}$$

From the previous equation it is clear that in order to force the limit to be finite and different from 0, the relative order of the denominator and numerator must be one ($n = m + 1$).

Combining this requirement with the requirements of the first propriety describe, it is easy to conclude that the minimum order function is second order, with the following format:

$$\tilde{K}_{ij}^{min}(s) = \frac{p_1 s}{s^2 + q_1 s + q_0},$$ <div align="right">**17**</div>

FINAL TIME VALUE

The response of a stable system to an impulse should tend to zero when time tends to infinite. This propriety establishes the bounded-input bounded-output stability of the radiation system and it is given by the limit:

$$\lim_{t \to \infty} K_{ij}(t) = \lim_{t \to \infty} \frac{2}{\pi} \int_0^\infty B(\omega) \cos(\omega t)\, d\omega = 0$$ <div align="right">**18**</div>

Therefore the poles of the transfer function $K_{ij}(s)$, zeros of the denominator $Q(s)$, must have a negative real part.

PASSIVITY

Passivity describes the property of systems that can store and dissipate energy, but not create it. Considering a floating body without external forces or incident waves, the *Cummins* equation can be written as

$$M\ddot{q}_i + C_{ij}^{hydrostatic} q = F_i^{radiation}$$ <div align="right">**19**</div>

The energy change of this system becomes

$$E(T) - E(0) = \int_0^T F_i^{radiation} \dot{q}_i dt$$ <div align="right">**20**</div>

And therefore the convolution term of the radiation force as to be passive. An interested reader can consult (Perez & Fossen, 2008) and the references there mentioned for a more detailed derivation.

For linear and time invariant systems passivity can be ensured if the retardation matrix is positive real define in the frequency domain:

$$\Re\{\tilde{K}_{ii}(s,\theta)\} = \Re\left\{\frac{P_{ii}(s,\theta)}{Q_{ii}(s,\theta)}\right\} > 0$$ <div align="right">**21**</div>

TABLE 1: PROPRIETIES OF RETARDATION FUNCTIONS

Propriety	Implications	Transfer function
1. $\lim_{\omega \to 0} K(j\omega) = 0$	There are zeros at s=0	$p_0 = 0$
2. $\lim_{\omega \to \infty} K(j\omega) = 0$	Strictly proper	$\deg\{Q(s,\theta)\} > \deg\{P(s,\theta)\}$
3. $\lim_{t \to 0} K(t) \neq 0$	Relative Degree 1	$\deg\{Q(s,\theta)\} - \deg\{P(s,\theta)\} = 1$
4. $\lim_{t \to \infty} K(t) = 0$	BIBO Stability	$\Re\{Q(s,\theta) = 0\} < 0$
5. The mapping $\dot{q} \to \mu$ is passive	$K(j\omega)$ is positive real	$\Re\{\tilde{K}_{ii}(s,\theta)\} = \Re\left\{\frac{P_{ii}(s,\theta)}{Q_{ii}(s,\theta)}\right\} > 0$

QUALITY OF THE MODEL

Assessing the quality of the model can be done in several ways. As described before, the frequency-domain model is evaluated using the frequency-response, while the time-domain models are evaluated by their impulse-response. In order to evaluate these responses, the R^2 value is computed using:

$$R^2 = 1 - \frac{\sum_l (K_{ij} - \widetilde{K}_{ij})^2}{\sum_l (K_{ij} - \overline{K}_{ij})^2}, \qquad 0 \leq R^2 \leq 1 \qquad \qquad \textbf{22}$$

were K_{ij} represents the reference retardation function, $\widetilde{K}_{ij}$ the parametric model and $\overline{K}_{ij}$ is the mean value of the reference retardation function. The summations are performed across all frequencies (for frequency response) or time (for impulse response).This is a measure of the amount of variability of the function that is captured by the model. The closer to one, the better is the quality of the fit.

FREQUENCY DOMAIN IDENTIFICATION METHODS

Providing the frequency-response of the convolution integral, computed with equation 7, one can fit a parametric model (transfer function) with the appropriate order for each entry of the retardation matrix:

$$\widetilde{K}_{ij}(s, \theta) = \frac{P(s, \theta)}{Q(s, \theta)} = \frac{p_m s^m + p_{m-1} s^{m-1} + \cdots + p_0}{s^n + q_{n-1} s^{n-1} + \cdots + q_0}$$

$$\theta = [p_m, \dots, p_0, q_{n-1}, \dots, q_0] \qquad \qquad \textbf{23}$$

The determination of the approximated model $\widetilde{K}_{ij}(s, \theta)$ and θ poses an optimization problem following the Least Squares Method:

$$\theta = \arg\min_\theta \sum_l \left(K_{ij}(s) - \widetilde{K}_{ij}(s, \theta) \right)^2 \qquad \qquad \textbf{24}$$

This problem is solved in the following the two methodologies presented in the next sections.

FREQ

This method was implemented in the work by Alves et al., 2011. The Least Squares (LS) Method is solved using the MatLab function *invfreqs* (Mathworks, n.d.). This function linearizes the optimization problem 24, using weight factors for the most important range of frequencies:

$$\theta = \arg\min_\theta \sum_l w_l \left(K_{ij}(s) - \widetilde{K}_{ij}(s, \theta) \right)^2 \qquad \qquad \textbf{25}$$

Where w_l is a user defined weighting vector with entries between 0 and 1 for each frequency. The linearization method used by the function *invfreqs* is based on the Levy method (Levy, 1959) and solved interactively (Sanathanan & Koerner, 1963). Equation 25 is re-written in the linear form:

$$\theta_p = \arg\min_\theta \sum_l w_l s_{l,p} \left(\widetilde{Q}_{ij}(s, \theta) K_{ij}(s) - \widetilde{P}_{ij}(s, \theta) \right)^2 \qquad \qquad \textbf{26}$$

Where

$$s_{l,p} = \frac{1}{\left| \widetilde{Q}_{ij}(s, \theta_{p-1}) \right|} \qquad \qquad \textbf{27}$$

The iterative method converges after $p = 10$ to 20 steps, when $\widetilde{Q}_{ij}(s, \theta_p) \approx \widetilde{Q}_{ij}(s, \theta_{p-1})$.

The order of the transfer function is determined using an automatic routine. The problem is initialized using a 2nd order function, with the form

$$\widetilde{K}_{ij}^{min}(s) = \frac{p_1 s + p_0}{s^2 + q_1 s + q_0},$$

<div align="right">28</div>

in order to guarantee the second propriety of Table 1 (see the section Proprieties of the parametric models). For each solution of the Least Squares Method, the quality of the fit is evaluated using the parameter R^2, calculated by equation 22. If the parameter R^2 is smaller than the user defined value, the order of both the denominator and numerator of the transfer function is increase, to fulfill the previously mention propriety. Once the transfer function is found, it is easy to determine the equivalent state-space model using the function *tf2ss*. Please refer to (Alves, et al., 2011) and (Alves, 2012) for more detail.

FDI TOOLBOX

The program incorporated the Frequency Domain Identification (FDI) Toolbox developed by (Perez & Fossen, 2009). This is a user-free toolbox and can be downloaded at www.marinecontrol.org.

This method is based on a similar approach as the one described in the previous section. However, three different methods are available to solve the optimization problem 25:

- The first method is equivalent to the one presented in the previous section, using a linearize LS minimization;
- In the second method an iterative linear LS problem is solved, using the as weight factors the previous denominator values;
- This last solves the non-linear LS problem, using a Gauss-Newton algorithm.

All the methods presented before use the function *invfreqs*. The iteratively linear LS problem provides the best computational time/accuracy relation and was the one chosen for this work according to the recommendations in Perez & Fossen, 2009. The toolbox uses the following algorithm in order to take into account the proprieties of the retardation functions described before Perez & Fossen, 2011:

1. Set the appropriate range of frequencies according to the user defined weight factors;
2. Scale the data:

$$\widetilde{K}_{ij}' = \alpha \widetilde{K}_{ij}; \quad \alpha = \frac{1}{\max \; |K_{ij}|}$$

<div align="right">29</div>

3. Select the order of the approximation $n = \deg\left(\tilde{Q}_{ij}(s, \theta)\right)$. The minimum order approximation $n = 2$ is the starting point.
4. Estimate the parameters θ using the iterative LS method, according to

$$\theta_p = \arg\min_\theta \sum_l \left| \frac{K_{ij}(s)}{s} - \frac{\tilde{P}_{ij}(s, \theta)}{\tilde{Q}_{ij}(s, \theta)} \right|$$

<div align="right">30</div>

5. Check stability by computing the roots of $\tilde{Q}_{ij}(s, \theta)$ (poles of the system) and change the real part of these roots with positive real part to a negative real part.
6. Construct the desired transfer function by scaling and incorporate the s factor in the numerator:

$$\widetilde{K}_{ij}' = \alpha \frac{s\tilde{P}_{ij}(s, \theta)}{\tilde{Q}_{ij}(s, \theta)}$$

<div align="right">31</div>

7. Estimate the added mass and damping based on the identified parametric approximation via

<div align="right">9</div>

$$\tilde{A}(\omega) = Im\{\tilde{K}_{ij}\} + A(\infty)$$
$$\tilde{B}(\omega) = \Re e\{\tilde{K}_{ij}\}$$

32

and compare with the $A(\omega)$ and $B(\omega)$ given by the 3d radiation/diffraction code. The quality of the fit is assessed using the parameter R^2, through equation 22 for the added mass and damping coefficients. If the fitting is not satisfactory increase the order of the approximation and go back to step (3).

8. Check for passivity if required $\Re e\{\tilde{K}_{ii}(s, \theta)\} > 0$

Step 4 of the procedure ensures that the first propriety of Table 1 is fulfilled. The second and third proprieties are fulfilled ensuring that the relative order of the functions is always one. The stability of the system is forced in step 5, and the passivity is verified in step 8. So this method ensures that most of the proprieties in table 1 are met, incorporating this *a priori* knowledge in fitted functions. This ensures a more accurate transfer function with a lower order. From the transfer functions it is once again easy to obtain the state-space model (*tf2ss.m*).

TIME DOMAIN METHODS

The identification of the state-space models can have as reference the impulse-response function of the retardation matrix, as presented in Figure 3. The conversion to time domain adds an additional error into the fitting method, due to the IFFT transformation. However this can be minimize, depending on the method to convert $K_{ij}(\omega)$ into time domain. One can compute the time-domain retardation function using:

$$K_{ij}(t) = IFFT\left(K_{ij}(\omega)\right)$$

33

This method is however limited by the Nyquist frequency. Due to the limited range of frequency usually used in the numerical codes, the discretization of $K_{ij}(t)$ will be evenly spaced and computed from 0 to high values of t. This means that there will be fewer points describing the functions for low values of t, where the impulse-response $K_{ij}(t)$ has a higher magnitude.

An alternative method to compute the impulse-response function of the retardation matrix is to use the cosine transformed described in equation 10. This was implemented using a trapezoidal integration method, as described in (Kristansen & Egeland, 2003):

$$K_{ij}(t) = \frac{\Delta\omega}{\pi} \sum_{k=1}^{k_{max}-1} 2B_{ij}(k\Delta\omega)\cos(k\Delta\omega t) + \frac{\Delta\omega}{\pi}\left(B_{ij}(0) + B_{ij}(k_{max})\cos(k_{max}\Delta\omega t)\right)$$

34

Where k_{max} is number of entries of the frequency vector computed by the numerical code. The step size used is determine by the length of the frequency vector, which is equally spaced using 256 points (e.g. for $k_{max}\Delta\omega = 5 rad/s$, $\Delta\omega \cong 0.02$). The upper limit is taken to be $T = 100s$, and the time step used was $\Delta t = 0.1s$.

This last method is used in the SS_Fitting code to compute the impulse-response function, necessary to implement the following time domain methods.

LEAST SQUARES METHOD

This method relies on a LS method to determine the realization of the retardation function. This method was also implemented in the work by Alves, et al., 2011.

It is based on the MatLab function *prony*. This function uses the *z-transform* to find an approximation to the impulse response function based on a combination of exponential functions in the time domain. The function returns the coefficients of numerator $b[k]$ and denominator $a[l]$ of the discrete rational system

$$H(z) = \frac{\sum_{k=0}^{q} b[k]z^{-k}}{1 + \sum_{l=0}^{p} a[l]z^{-k}}$$

<div align="right">35</div>

From the transfer functions it is once again easy to obtain the state-space model (*tf2ss.m*).

The discrete transfer function needs to be converted to the continuous time domain, using the function *d2c*, with the Tustin method. However, for complex high order retardation functions, this not ensures the stability of the resulting state-space model.

The order of the transfer function is determined assessing the quality of the fit using the R^2 value, as described previously. If the model does not fulfill the minimum required user defined value, the method is run again using a higher order rational function.

REALIZATION THEORY

Once the impulse-response function is obtained using 34, the identification scheme based on Hankel Singular Value Decomposition (SVD) is applied. This method was proposed by Kung, 1978 and is available in the MatLab function *imp2ss*. For a detailed description of the SVD method one should consult (Kung, 1978).

The method outputs the equivalent state-space system, $\overline{A_{\iota J}}$, $\overline{B_{\iota J}}$, $\overline{C_{\iota J}}$ and $\overline{D_{\iota J}}$, which need to be scaled according to the time step used in $K_{ij}(t)$:

$$A_{ij} = \overline{A_{\iota J}}, \qquad B_{ij} = \overline{B_{\iota J}}, \qquad C_{ij} = \overline{C_{\iota J}}\Delta t \quad D_{ij} = \overline{D_{\iota J}}.0 = 0$$

<div align="right">36</div>

The matrix D_{ij} is forced to be zero, in order to keep the causality of the system. Despite the reduction option built-in the *imp2ss* function, this proves to be not satisfactory way to control the accuracy and order of the fitted model. The function produces very accurate models ($R^2 > 0.99$), using however with a very high orders ($n > 200$). However, the computations of the Hankel singular values revealed that only a small number of states have a significant energy value (e.g. Figure 4). In the example it is clear that the first two singular values have an absolute value much higher than all the others. In fact, this function can be approximated with a second order system with $R^2 > 0.98$.

In order to obtain a low order model, the reduction of the number of states was implemented using the function *balmr*. This function can be applied using two methods. Using the manual method, the user chooses the number of states to keep, based on the Hankel Singular Values plot. Moreover an automated method was also implemented, using the goodness of the fit R^2 calculated with equation [22], for the impulse-response function. The method reduces the number of states to a second order function, and then increases the order of the system until the user-defined goodness is achieved.

FIGURE 4: HANKEL SINGULAR VALUES OF THE IMPULSE-RESPONSE FUNCTION SURGE-SURGE FOR THE OC3HYWIND SPAR BUOY.

MATRIX ASSEMBLY

Using the above described methods, a set of state-space systems are obtained, one for each significant entry of the retardation matrix K. We obtain several state-space systems, according to:

$$\mu_{ij} = \int_0^t K_{ij}(t-\tau)\dot{q}\delta\tau \cong \begin{vmatrix} \dot{x}_{ij} = A_{ij}x_{ij} + B_{ij}\dot{q}_J \\ \mu = C_{ij}x_{ij} \end{vmatrix} \qquad \textbf{37}$$

where the subscripts i and j vary from 1 to m, where m is the number of rigid-body platform degrees of freedom enabled (up to 6). This equation and the others below do not follow Einstein notation. The retardation matrix K is a m by m matrix. For most of the floating bodies, only the main diagonal and some off-diagonal terms of the retardation matrix are non-negligible. The size of each matrices A_{ij}, B_{ij} and C_{ij} are respectively $[n_{ij} \times n_{ij}]$, $[n_{ij} \times 1]$ and $[1 \times n_{ij}]$, where n_{ij} is the number of states used to approximate the entry K_{ij}. These matrices may or may not be full depending on the identification method. The size of x_{ij} is $[n_{ij} \times 1]$.

In order to obtain the complete state-space system, each of the matrices A_{ij}, B_{ij} and C_{ij} have to be assembled into a global state-space system according to the following equations:

$$\dot{x}_r = A_r x_r + B_r \dot{q}$$
$$\mu = C_r x_r$$

$$A_r = [n \times n] = \begin{bmatrix} [A_{11}] & & & & & & & & & \\ & \ddots & & & & & & & & \\ & & [A_{1m}] & & & & & & & \\ & & & [A_{22}] & & & & & & \\ & & & & [A_{21}] & & & & & \\ & & & & & \ddots & & & & \\ & & & & & & [A_{2m}] & & & \\ & & & & & & & \ddots & & \\ & & & & & & & & [A_{mm}] & \\ & & & & & & & & & [A_{m1}] \\ & & & & & & & & & & \ddots \\ & & & & & & & & & & & [A_{mm-1}] \end{bmatrix}, \qquad \textbf{38}$$

$$B_r = [n \times m] = \begin{bmatrix} [B_{11}] \\ \vdots \\ [B_{1m}] \\ & [B_{22}] \\ & [B_{21}] \\ & \vdots \\ & [B_{2m}] \\ & & \ddots \\ & & & [B_{mm}] \\ & & & [B_{m1}] \\ & & & \vdots \\ & & & [B_{mm-1}] \end{bmatrix},$$

$$C_r = [m \times n] = -\begin{bmatrix} [C_{11}] & & & [C_{21}] & & & [C_{1m}] \\ & [C_{12}] & \ddots & [C_{22}] & & & & \ddots \\ & & & & [C_{23}] & \cdots & & & \\ & & & & \ddots & & & [C_{mm-1}] \\ & [C_{1m}] & & & [C_{2m}] & [C_{mm}] & & \end{bmatrix},$$

$$x_r = [n \times 1] = \begin{bmatrix} [x_{11}] \\ \vdots \\ [x_{1m}] \\ [x_{22}] \\ [x_{21}] \\ \vdots \\ [x_{2m}] \\ [x_{mm}] \\ [x_{m1}] \\ \vdots \\ [x_{mm-1}] \end{bmatrix} \qquad \dot{q} = [m \times 1] = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_m \end{bmatrix}$$

where n describes the total number of radiation states, and m the number of platform dof's. The matrices are ordered by the first subscript, and the diagonal term $[x_{ii}]$ always appears first , followed by the other cross terms with the same index $[x_{ij}]$. Note that the minus sign on the matrix C_r accounts for the minus sign on the memory effects on equation 3.

FAST INTEGRATION

As described in the introduction, SS_Fitting enables the use of the new FAST state-space module: *SS_Radiation* (Duarte & Jonkman, 2012). Therefore this routine should be used as a pre-processor, in order to generate the required "name.ss" input files, containing the matrices A_r, B_r and C_r.
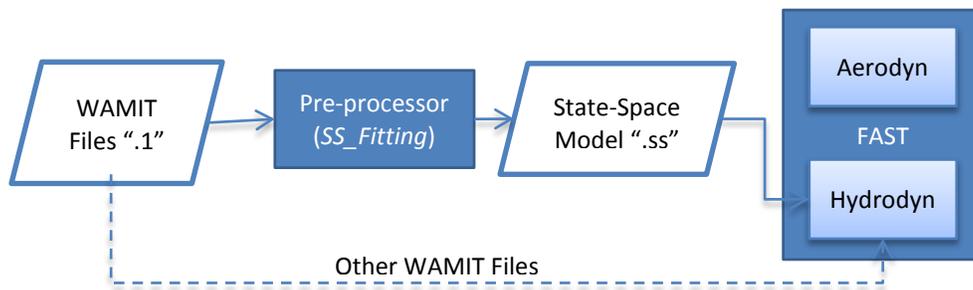

FIGURE 5: MATLAB PRE-PROCESSOR AND REQUIRED FILES.

USER GUIDE

RETRIEVING FILES FROM THE ARCHIVE

You can download the ss_fitting archive from our web server at *http://wind.nrel.gov/designcodes/postprocessors/SS_Fitting*. The file has a name similar to *SS_fitting_v1.00.00.exe*, but may have a different version number. Create an SS_fitting folder somewhere on your file system and put this file there. When you double click on the archive from Windows Explorer, it will create some files and folders. To use the scripts, you need to add SS_Fitting's *Source*, folder to the MatLab search path.

DISTRIBUTED FILES

SS_Fitting includes the following files:

Change_log.txt	Log file with the updates of the current version
Disclaimer.txt	Disclaimer file
SS_Fitting_Manual.pdf	Theory and User Manual (this document)
SS_Fitting_Options.inp	Sample input file with user defined options
Source/SS_Fitting.m	Main MatLab File
Source/Radiation.m	MatLab routine responsible to fit the retardation function models
Source/FDI_Toolbox/**	Folder containing the adapted files from the FDI toolbox[1]
Verification/Verification_Routine.m	Verification routine with comparisons of the several methods
Verification/Inertia.m	Used by the verification routine
Verification/Hydrostatic.m	Used by the verification routine
Verification/spar_097/**	Folder containing the results files using the OC3Hywind spar buoy as reference platform, and R^2=0.97
Verification/spar_099/**	Folder containing the results files using the OC3Hywind spar buoy as reference platform, and R^2=0.99
Verification/marin_semi_097/**	Folder containing the results files using the OC4 Semi-submersible as reference platform, and R^2=0.97

RUNNING SS_FITTING

To run SS_Fitting it is require to have a *MatLab* license and access to the *Control Systems toolbox*. Depending on the method chosen, additional toolboxes are required. The frequency domain methods use the *invfreqs* function, available in the *Signal Processing Toolbox.* The time-domain Least Squares method uses the function *prony*, available in the *Control Systems toolbox.* Finally to use the time-domain realization theory method, the user must have a license to the *Robust Control Toolbox*.

Before running SS_Fitting, make sure that SS_Fitting *Source* is in the MatLab search path. The user should define all the inputs using the input file "*name.inp*". Please refer to the following section for details on the input file.

Write *ss_fitting('name.inp')* on the MatLab command window. *'name.*inp' can include the absolute or relative path. The program should run and the chosen fitting method should appear on the screen. If the program runs successfully the message "Results saved in *Location* …" should appear on the command window. The results were written in an ACSIS file call "*platform_name.ss",* within the WAMIT file location. This contains the state-space matrices A_r, B_r, and C_r of the radiation state space model.

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[1] (Perez & Fossen, 2009) www.marinecontrol.org

THE INPUT FILE

The input file determines the program options, including the method used to compute the fitting. Please use the supplied example file as a reference. The file must contain the following lines:

Option	Variable Name	Description
Header Line	-	Line containing the file title
File location	*Rad.FileName*	Base name and location of the WAMIT files for the desired platform. The location can be absolute or relative to the SS_Fitting folder.
DoF Vector	*gp.DoF*	[1x6] Vector containing 0 or 1 if the correspondent platform degree of freedom is enabled or not. It follows the order Surge, Sway, Heave, Roll, Pitch and Yaw. For a 6 DoF platform use [1,1,1,1,1,1].
Frequency Range	*Rad.twr*	Typical range of frequencies that appear during time-domain simulation. This most important range will be weighted in the fitting with the weight factor defined below.
Weight factor	*Rad.wwf*	Weight factor for the defined frequency range, between 0 and 1. If 1 is chosen than only the frequencies within the defined range are evaluated for the goodness of the fit, neglecting frequencies outside the defined range.
Identification Method	*Rad.tfi*	Identification method to be used: 1. Freq. Domain Identification 2. FDI Toolbox 3. Time-Domain LS method 4. Time-Domain Realization theory[2]
Fit Accuracy	*Rad.fit*	Minimum R^2 value to be satisfied by the fit. Use this value to control the accuracy and order of the fit. It should be between $0.95 < R^2 < 0.99$.
Plot flag	*Rad.ppmf*	Enable (0 or 1) to plot the retardation response. Either in the frequency- or time-domain according to the method chosen.
Reduction Mode flag	*Rad.fmt*	(0 or 1) to chosen use a manual or automatic order reduction scheme. Only used with Time-Domain Realization theory method.

Please refer to the previous sections for details about the different methods implemented.

FIGURE 6: SAMPLE INPUT FILE.

OUTPUT FILE

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[2] Please refer to (Duarte, et al., 2013) for more details on the quality of the methods.

The output file contains the matrices A_r, B_r and C_r of the state-space system:

$$\dot{x}_r = A_r x_r + B_r \dot{q}_{plat}$$
$$\mu = C_r x_r$$

<div align="right">39</div>

with the following dimensions:

$$A_r = [n \times n], \quad B_r = [n \times m], C_r = [m \times n], \quad x_r = [n \times 1]$$

<div align="right">40</div>

Where n is the number of radiation states and m is the number of dof's enabled. The file also contains a header line, containing the program version and the date in which it was run. It also contains the enabled DoF's, the number of states, and the number of states per degree of freedom. These variables are used by the FAST *SS_Radiation* module. In order to run FAST using this module, the output file *.ss should be present in the same folder of the other WAMIT files.

FIGURE 7: SAMPLE OUTPUT FILE.

VERIFICATION ROUTINE

The archive includes a MatLab routine that plots the results of the different methods for different platforms. To run the script, write *Verification* in the MatLab command window, making sure that the folder *Verification* is the MatLab current folder. Three tests are available:

spar_097	Comparison for the OC3Hywind spar buoy using R^2 value equal to 0.97
spar_099	Comparison for the OC3Hywind spar buoy using R^2 value equal to 0.99
Marin_semi_097	Comparison for the OC4 Marin semi-submersible platform, using R^2 value equal to 0.97

The routine plots for the different significant entries of the retardation matrix, the frequency domain and time domain response functions and the recalculated added mass and damping matrices for each method, from equation 7.

From the results shown and further discussed in (Duarte, et al., 2013), we recommend that the user chooses between the FDI or the Realization theory methods.

FUTURE WORK

Future work will include the verification of the results and comparison of the different methods. Some of them maybe removed from the program if the results are not satisfactory.

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FEEDBACK

If you have problems with SS_Fitting, please contact Tiago Duarte. Please send your comments or bug reports to:

Tiago Duarte (Email: tduarte@hidro1.ist.utl.pt)

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